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By

Robert L. Ash

Progress Report

Prepared for the
National Aeronautics and Space Administration
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Robert L. Ash[1]

SUMMARY

A computer program has been developed to simulate the transient wall pressure field produced by a low speed fully turbulent boundary layer. The theoretical basis for the simulation has been discussed and preliminary results from a pressure simulation are presented.

INTRODUCTION

The purpose of this report is to present a method for simulating the unsteady pressure fluctuations produced by a low speed incompressible turbulent boundary layer. Attention has been restricted to flow along flat plates with no pressure gradient. The simulation has been designed for utilization in analysis of compliant wall candidates for the Langley Research Center drag reduction program (refs. 1 through 5). A one-dimensional simulation is presented here which is compatible with both finite difference and finite element structural programs. Two-dimensional simulations can be developed by randomly accessing the one-dimensional simulation, but that development is not presented here.

Properties of the turbulent simulation have been extracted primarily from three experimental papers. The very extensive early work of Bull (ref. 6) has been employed to model the frequency spectrum, the convection velocity, and the decay of pressure fluctuations. Burton's (ref. 7) measurements have been used to

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justify the assumption that turbulent sublayer "burst" data (see ref. 8 for a survey of basic sublayer phenomena) can be used to infer spatial and temporal distributions of pressure fluctuations in the outer portion of the boundary layer. (This inference does not necessarily imply that the burst structures ultimately form large scale pressure fluctuations.) Burton's work has also been used to estimate the distribution of pressure fluctuation amplitudes occurring in a collection of events. Offen and Kline's (ref. 9) burst distribution measurements have been used to model the spatial and temporal distribution of the pressure fluctuations.

In order to minimize the influence of numerical resolution on the simulation, a random stepping procedure has been employed in space and time. The magnitude of the pressure fluctuation at a particular point at any time is constructed from a collection of the randomly generated pressure fluctuation "events". Each of the events which contribute to the local pressure level has been constructed to preserve the turbulence statistics measured in experiments. Probability distributions for time between events, distance between events, fluctuation frequency, and amplitude have been employed. Each probability distribution has been assumed statistically independent from the others, and a conventional Monte-Carlo approach has been used. Some detail is provided in the following section.

MONTE-CARLO METHOD FOR SIMULATING WALL PRESSURE FREQUENCY SPECTRUM

Bull (ref. 6) has reported the wall pressure frequency spectrum shown in figure 1 for a low speed turbulent boundary layer. He has shown his measurements are well represented by the empirical relation:

$$\phi_p(\omega) = \frac{q_\infty^2\ \delta^*}{U_\infty} \times 10^{-5} \left[3.7e^{-2\frac{\omega\delta^*}{U_\infty}} + 0.8e^{-0.47\frac{\omega\delta^*}{U_\infty}} - 3.7e^{-8\frac{\omega\delta^*}{U_\infty}} \right] \tag{1}$$

where q_∞ is the dynamic pressure, δ^* is the displacement thickness, U_∞ is the free stream velocity, and ω is the radian frequency.

By assuming equation (1) adequately describes the frequency distributions of the pressure fluctuations occurring in a large number of events, the probability of a particular frequency occurring in a particular event can be calculated. That probability would simply be the normalized form of equation (1) or:

$$P(\omega) = \phi_p(\omega) / \int_0^\infty \phi_p(\omega)\, d\omega \quad . \tag{2}$$

The probability distribution cannot be used directly in a simulation. Rather, the cumulative probability distribution must be used. The cumulative probability is defined as the probability an event will occur with a frequency less than or equal to ω, whereas probability was the likelihood an event would occur between ω and $\omega + d\omega$. Probability $P(\omega)$ and cumulative probability $\hat{P}(\omega)$ are related by

$$\hat{P}(\omega) = \int_{-\infty}^{\omega} P(\omega) d\omega \tag{3}$$

However, in this case, $P(\omega)$ is zero for all values of ω less than zero, leaving:

$$\hat{P}(\omega) = \int_{0}^{\omega} P(\omega) d\omega \quad . \tag{4}$$

Equation (1) can be employed in (2) and (4) to write:

$$\hat{P}(\omega) = 1 - 0.5916e^{-2\lambda} - 0.5443e^{-.47\lambda} + 0.1359e^{-8\lambda} \tag{5}$$

where $\lambda = \frac{\omega \delta^*}{U_\infty}$. Dimensionless frequency is shown as a function of cumulative probability in figure 2.

Most digital computers have available efficient random number generation subroutines. Generally, they produce uniformly distributed random numbers over the interval between zero and unity. If values of frequency are assigned from figure 2, using randomly generated numbers between zero and one, the frequencies will be distributed in agreement with Bull's (ref. 6) data. A brief explanation of the basis for this approach is given in reference 10.

Graphical procedures are not acceptable for efficient computer utilization. Furthermore, because equation (5) is a transcendental equation for λ a function of $\hat{P}$, it cannot be used efficiently in calculating λ for a given random number. Since equation (5) is itself an empirical curve fit, this investigation has employed an approximate equation for λ a function of $\hat{P}$ to streamline computer calculations. The equation:

$$\lambda = 0.2173 \sqrt{\hat{P}_\lambda} - 0.3070 \hat{P}_\lambda + 0.7899 \hat{P}_\lambda^2 + 3.3518 \left[\frac{1}{(1 - \hat{P}_\lambda)^{1/4}} - 1 \right] \tag{6}$$

appears to satisfactorily represent equation (5), as shown in figure 2. Detectable errors occur only on the upper end of the plot between 0.8 and unity, with a maximum error of about 10 percent at $\hat{P} = 0.9$. That error is within the uncertainty of Bull's (ref. 6) data fit.

The same procedure has been used to generate the other turbulent characteristics simulated here. The remaining discussion will only document the appropriate probability distributions and their representations in the computer simulation.

Amplitude of a Pressure Fluctuation Event

Experimental measurements of the amplitude of a pressure fluctuation produced by a single fluctuation event is not currently possible because background contributions from other events are always present. Such an amplitude is required in the present simulation. The only data which is related (indirectly) to individual fluctuation amplitudes are the measurements of Burton (ref. 7) for the threshold pressure fluctuation which appeared to be responsible for a burst event. His measurements suggest the distribution of amplitudes is Gaussian. Since Bull's (ref. 6) measurements indicate the root mean square (rms) pressure fluctuation is given by $p'_{rms} = 3\tau_\omega$, at the speeds of interest, a Gaussian distribution can be employed using a standard deviation of $3\tau_\omega$ about a zero average pressure. The Gaussian distribution was simulated using conventional techniques (ref. 11).

Time Interval and Spacing Between Pressure Fluctuation Events

Offen and Kline (ref. 9) have studied the time interval between burst events in the wall region of a low speed turbulent boundary layer. They have found that the time interval can be scaled with outer flow variables and the probability of a new event varies with dimensionless time θ given by $\theta = U_\infty t/\delta^*$, as shown in figure 3. That data can be represented by gamma distribution function:

$$p(\theta) = \theta^\alpha \, e^{-\theta/\beta} / \left[\Gamma(\alpha + 1)\ \beta^{\alpha+1}\right] \tag{7}$$

where $\alpha = 2.2$ and $\beta = 16.4$, as shown in figure 3.

Equation (7) was integrated numerically to obtain the cumulative probability distribution $\hat{P}(\theta)$ shown in figure 4. That distribution has been approximated by:

$$\theta = 32.2 - \frac{2}{\hat{P}_\theta + 0.619} + 72\ \hat{P}_\theta^2 + 0.63 \tan \frac{\pi}{2} \hat{P}_\theta \tag{8}$$

and is also shown in figure 4.

The spatial distance between events can be gotten by assuming time and space are related through the friction velocity u_τ. Examination of Offen and Kline's (ref. 9) data has suggested that the Δx and Δt for spacing between events are related by:

$$\frac{\Delta x}{\delta^*} = \frac{u_\tau \Delta t}{\delta^*} = \frac{u_\tau}{U_\infty} \Delta\theta \quad \text{or} \quad \Delta x = \delta^* \frac{u_\tau}{U_\infty} \Delta\theta \ . \tag{9}$$

Consequently, random numbers can be used in equation (8) to generate x where

$$x = \delta^* \frac{u_\tau}{U_\infty} \theta \tag{10}$$

is the spacing between fluctuation events.

Convection Velocity and Decay Rate

Data from Bull's experiments (ref. 6) have been used to model both convection velocity and decay rate. Although Bull reports the convection velocity varies with distance from the fluctuation source, a constant value of convection velocity, u_c, given by

$$u_c = 0.8\ U_\infty\ , \tag{11}$$

has been used in the present simulation. The constant assumption was made because of difficulties in the numerical calculations, but may be justified from other experimental data [see Willmarth (ref. 12) for example].

Bull's experiments (ref. 6) indicate spatial decay scales with $x^+ = \frac{xu_\tau}{\nu}$. His data has been used to vary the amplitude of the pressure fluctuation in a particular event. If A_o is the original amplitude, $A(x^+)$ has been modeled by:

$$A(x^+) = A_o \left[1 - e^{-\frac{4267}{x^+}} \right] \tag{12}$$

Both convection velocity and decay rate have been "modeled" rather than "simulated" in the sense that all events are assumed to have the same convection and decay properties. At this point equations have been developed to simulate frequency, amplitude, and spacing of pressure fluctuation events. The remaining discussion is intended to explain how these models have been put together to simulate a turbulent wall pressure.

SIMULATION OF THE TURBULENT WALL PRESSURE

The ultimate output of this simulation is to be a simulated pressure field over a specific distance (test model) with a specific time interval. In order to generate that simulation, both a development length and a start-up time are required. Theoretically, disturbances extending upstream to infinity can contribute to local pressure fluctuations which implies that the start-up time would also be infinite. Practically, a finite start-up length can be used which accounts for nearly all of the local pressure fluctuation. However, because of the random time stepping procedure employed here, the start-up time had to be determined by numerical testing.

The decay rate defined in equation (12) can be used to specify the development length. If fluctuations less than one percent of their original value are neglected, then the start-up length is given by:

$$x^+ = 425{,}000 = \frac{xu_\tau}{\nu} \tag{13}$$

If $u_\tau = 1.5$ m/sec and $\nu = 1.5 \times 10^{-5}$ m²/sec, the start-up length is 4.25 m.

The procedure employed here has been to define the origin ($x = 0$) as the front of the start-up length. Assuming the start-up length is x_D, the front of the model would be at $x = x_D$. Using random numbers, frequency f, amplitude P_o, origin Δx, and time Δt are generated for a particular disturbance using the previously described

equations. The amplitude of the pressure fluctuation can be positive or negative; all other quantities are positive.

A single-cycle, sinusoidal pressure fluctuation represents a single event. If x_p is the location of the previous pressure fluctuation, the new fluctuation is located at

$$x = x_p + \Delta x \quad . \tag{14}$$

The time over which the event is sensed at a particular spatial location T is given by:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \tag{15}$$

Using the convection velocity u_c the spatial length of the event is given by:

$$W_D = u_c T \tag{16}$$

and the origin of the disturbance is assumed to occur at

$$x_o = x - W_D = x_p + \Delta x - u_c T \quad . \tag{17}$$

The time of birth of the event, t_b, is assumed given by

$$t_b = t_o + \Delta t \tag{18}$$

where t_o is a reference time which will be discussed later.

If the disturbance is assumed "born" instantly over the interval between x_o and x, at time t_b, the distance traveled by the disturbance at some later time t is given by:

$$D = u_c \cdot (t - t_b) \tag{19}$$

and the decayed amplitude $P_a(t)$ is:

$$P_a(t) = P_o \left[1 - e^{\frac{4267\nu}{u_c u_\tau (t-t_b)}} \right] \tag{20}$$

If x_s is a particular location on the test surface where the pressure history is desired, then the arrival time for the disturbance is:

$$t_a = t_b + \frac{x_s - x}{u_c} \tag{21}$$

and the departure time is:

$$t_d = t_b + \frac{x_s - x_o}{u_c} \tag{22}$$

Furthermore, if t is any time in the interval, $t_a \leq t \leq t_b$, the pressure produced at x_s by that particular event is:

$$P'(x_s, t) = P_a(t) \sin 2\pi \frac{t - t_a}{\tau} \tag{23}$$

Equation (23) then represents the pressure contribution at desired location x_s at time t and is the basis for all further calculations.

Assuming reference time t_o is known, pressure fluctuations of the type just described can be generated at a random set of points, starting at $x = 0$ and stepping to $x = x_D + x_m$, where x_m is the model length, using randomly generated Δx's. In addition, times of birth can be distributed randomly above t_o by using random Δt's in equation (18). Then, using equation (23) the pressure contributions at particular points can be calculated.

If Δt_a is defined as the average time step generated in stepping from $x = 0$ to $x = x_D + x_m$, when the end of the model is reached, calculations can return to the origin and step down

the model again. The reference time would then be updated as:

$$t_o = t_o + \Delta t_a \tag{24}$$

That process can be repeated over and over until a particular time is reached ($t_o = t_{max}$). Assuming that part of the simulation is completed, two problems still remain--how to specify to and how to store the simulation.

It is reasonable to assume the pressure simulation should be over the time interval $0 \leq t \leq t_{max}$. However, t_o cannot be set equal to zero to start with, because start-up transients would be present in the simulation. Since the time required for a disturbance to travel from $x = 0$ to $x = x_D + x_m$ is $\frac{x_D + x_m}{u_c}$ a logical start-up time would be:

$$t_{min} = \frac{x_D + x_m}{u_c} \tag{25}$$

However, t_{min} should be larger because some of the disturbances may occur upstream from $x = 0$, i.e., x_o can be negative. A conservative value for start-up time given by

$$t_o = -1.44\ t_{min} \tag{26}$$

has been used in the present simulation. Then at $t = 0$, start-up transients should no longer exist.

Locations and time steps used in the storage of the simulated pressure have no effect on the calculations and are specified based on the spatial and time resolutions for the structural calculations. An x_s array can be specified, as well as the desired structurally compatible time step Δt_s. Then, if a single disturbance passes over x_s between t_a and t_d, integer start number N_{go} and stop number N_{stop} given by:

$$N_{go} = \frac{t_a}{\Delta t_s} + 0.99 \qquad \text{and} \qquad N_{stop} = \frac{t_d}{\Delta t_s} \tag{27}$$

can be used to assign storage time locations for $P'(x_s, t)$. That is, the pressure $P(x_s, t_n)$ has been increased by an amount $P'(x_s, t_n)$ for $t_n = n\Delta t_s$, $N_{go} \leq n \leq N_{stop}$. It is important to note that $P'(x_s, t_n)$ is not the pressure because more than one event may be over x_s at a given time.

A program for performing the pressure simulation just described is included in Appendix A. Output from a sample simulation run is shown in figure 5. It shows the variation of pressure with time at a single point. All aspects of the simulation have not been verified at this time. That is, output power spectra and correlations have not been examined and compared with experimental data. However, the root mean square pressure fluctuation and average pressure agree quite well with their desired values ($3\ \tau_\omega$ and 0, respectively).

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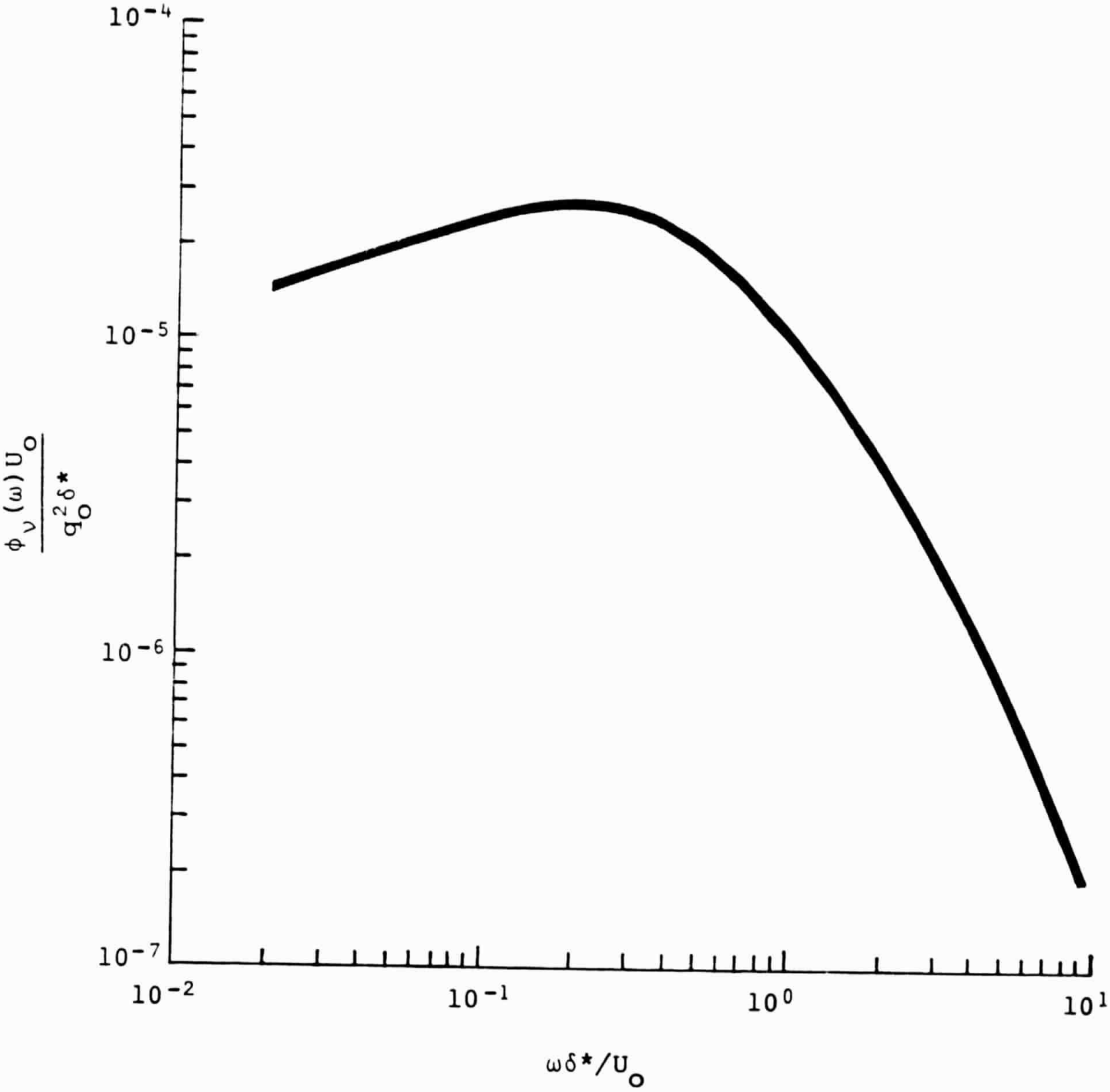


10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\frac{\phi_\nu(\omega)U_o}{q_o^2\delta^*}$
10^{-2}
10^{-1}
10^{0}
10^{1}
$\omega\delta^*/U_o$

Figure 1.

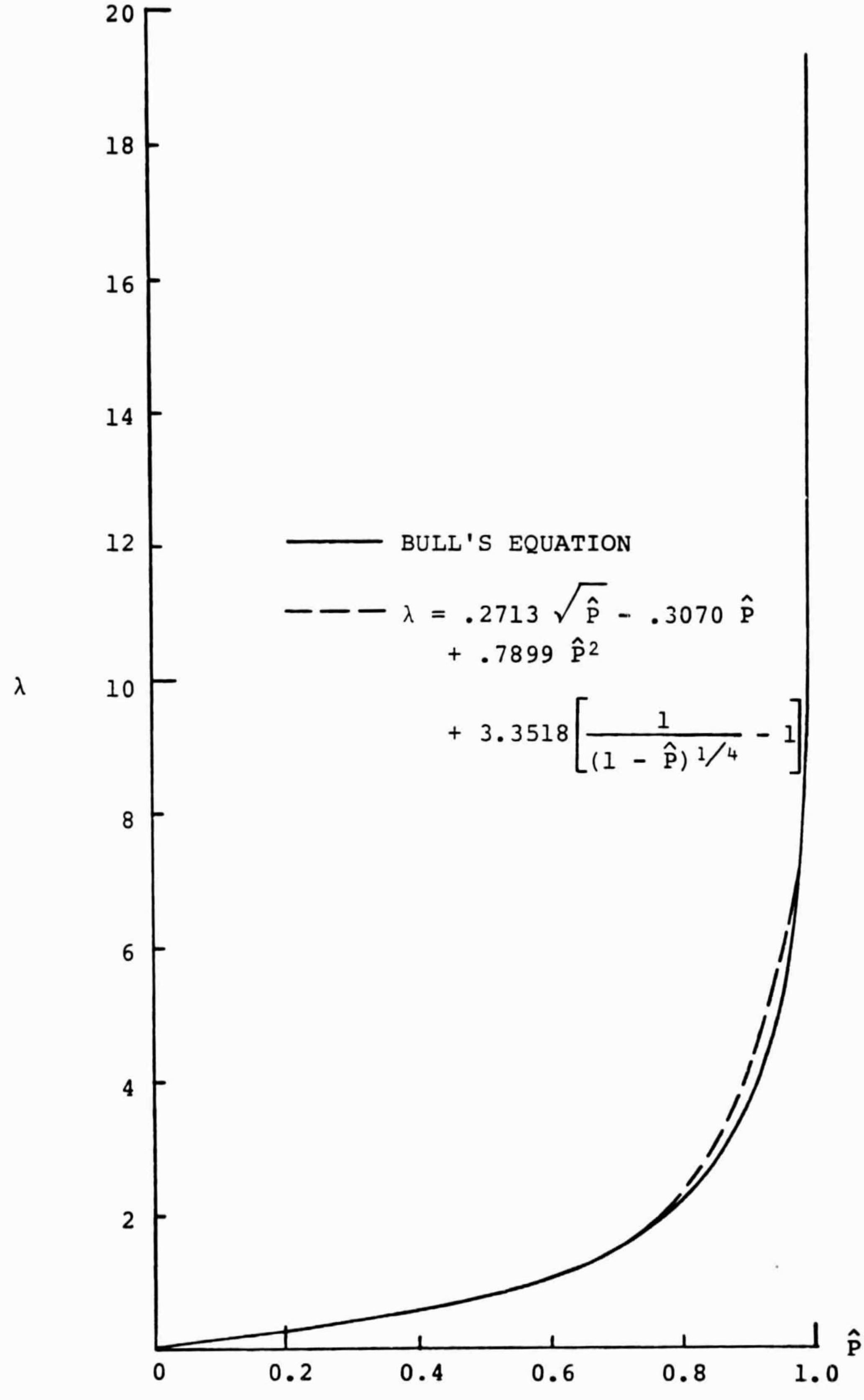
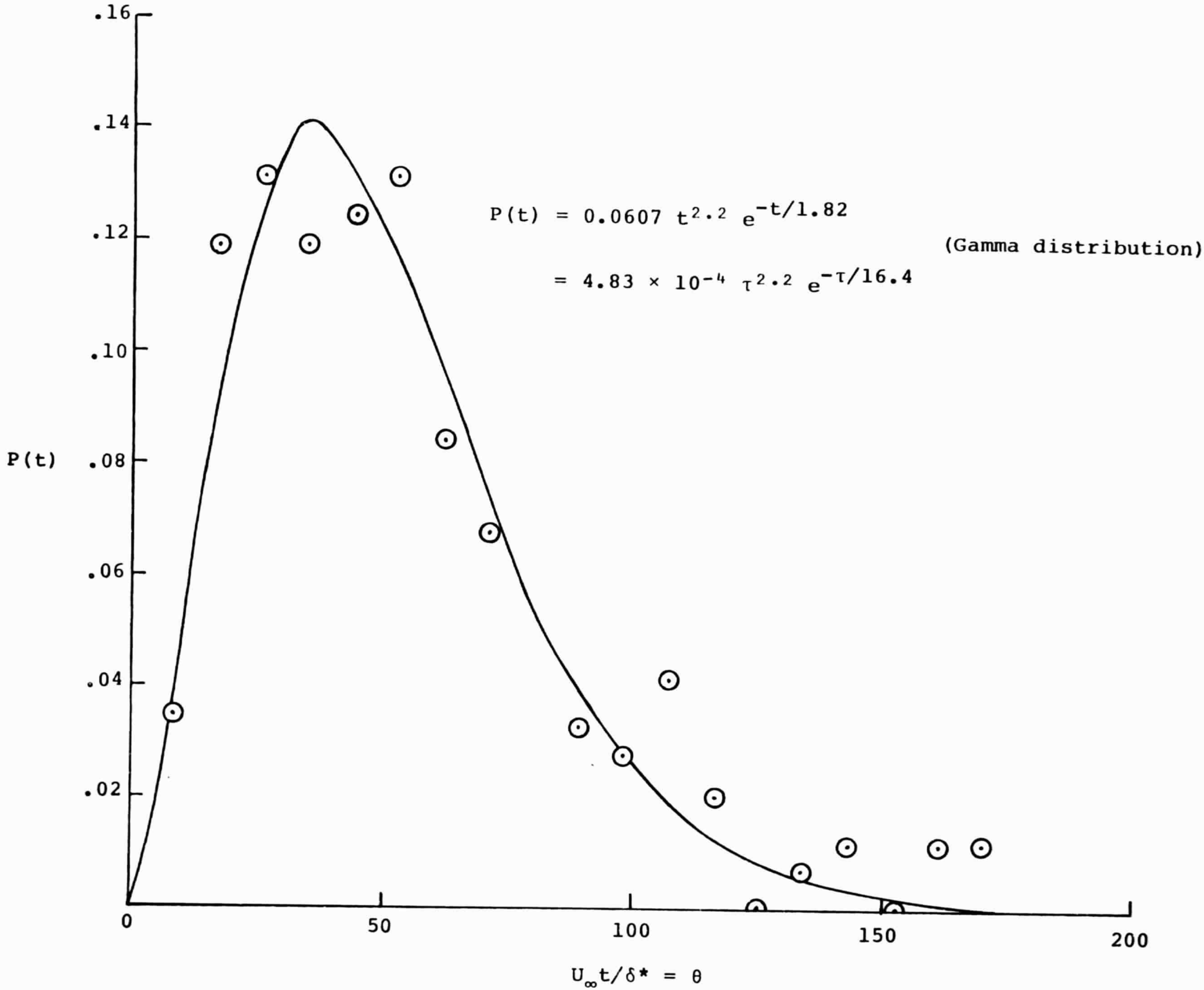
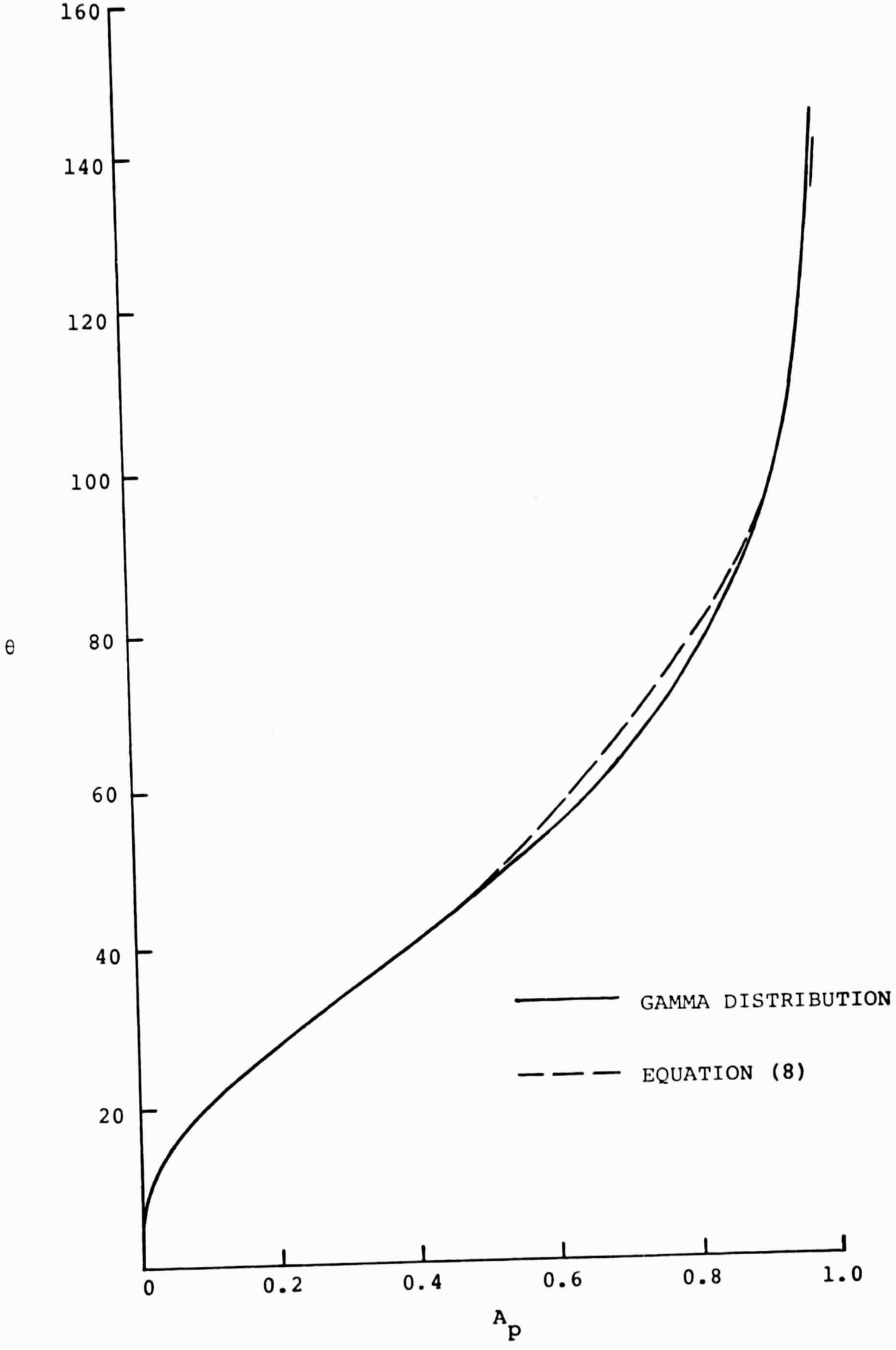

20
18
16
14
12
10
8
6
4
2
λ
BULL'S EQUATION
$\lambda = .2713\sqrt{\hat{P}} - .3070\,\hat{P} + .7899\,\hat{P}^2 + 3.3518\left[\frac{1}{(1-\hat{P})^{1/4}} - 1\right]$
0
0.2
0.4
0.6
0.8
1.0
$\hat{P}$

Figure 2.



$P(t) = 0.0607\ t^{2.2}\ e^{-t/1.82}$
$= 4.83 \times 10^{-4}\ \tau^{2.2}\ e^{-\tau/16.4}$
(Gamma distribution)
P(t)
.16
.14
.12
.10
.08
.06
.04
.02
0
50
100
150
200
$U_\infty t/\delta^* = \theta$

Figure 3.



160
140
120
100
80
60
40
20
θ
GAMMA DISTRIBUTION
EQUATION (8)
0
0.2
0.4
0.6
0.8
1.0
A_p

Figure 4.

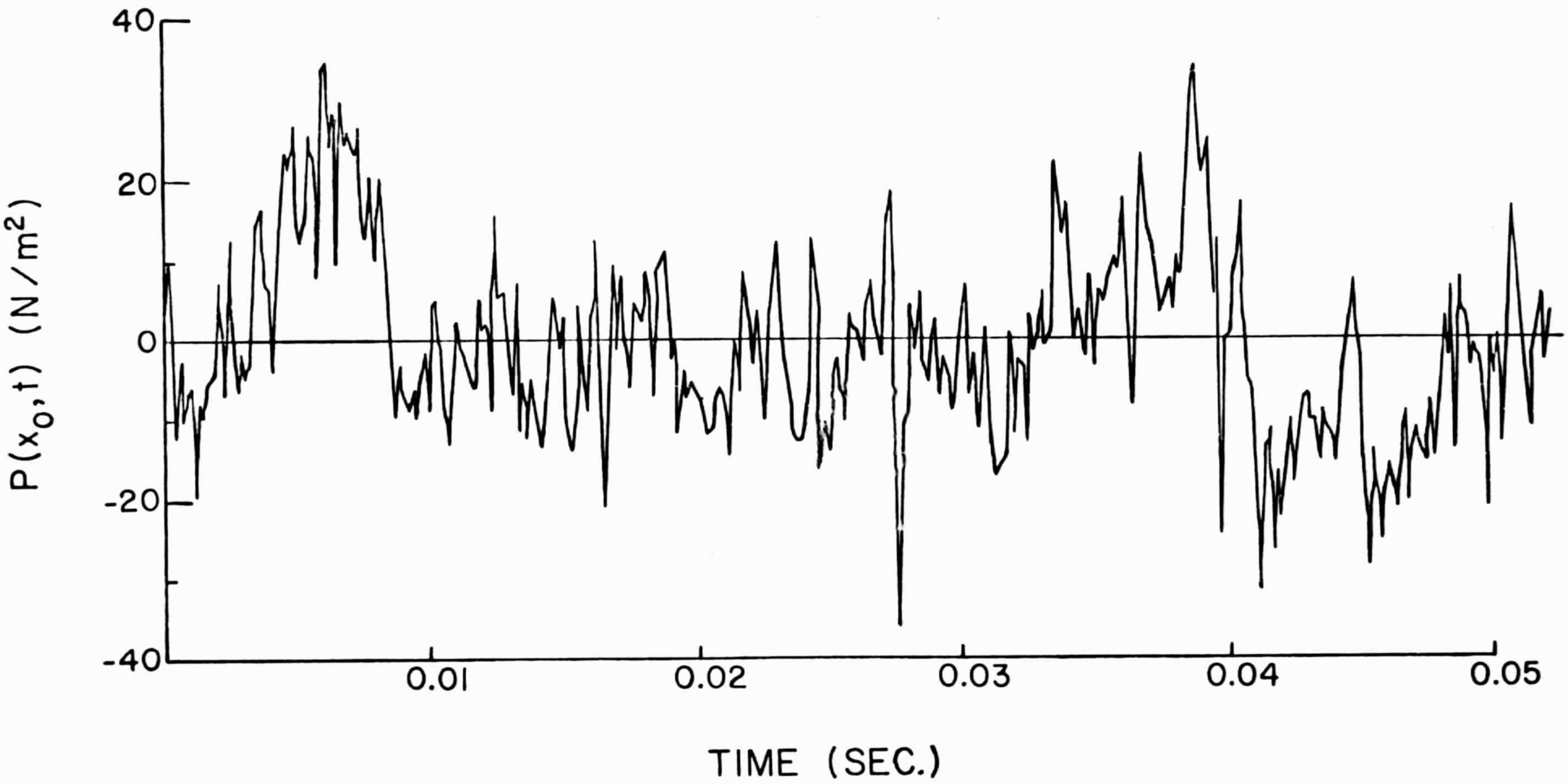


Figure 5.

APPENDIX A

FORTRAN PROGRAM FOR SIMULATING TURBULENT WALL PRESSURE

```
JOB.1.6000.125000.10000.         A4677    R4623        100718          BIN34
USER.BALA,RAMAKRISHNAN                     000029300F 37220 NAS          ODU
RUN(S)
REQUEST.TAPE1.HY.         3511017.RIL.RSB.TW20.
REWIND(TAPE1)
LGO.
-
      PROGRAM SIMU(INPUT.OUTPUT.TAPE5=INPUT.TAPE6=OUTPUT.TAPE1.TAPE2)
      DIMENSION P(2.10000)
      NR=5$NW=6
C     NDIM=MAXIMUM TIME DIMENSION
      NDIM=10000
      CDIM=NDIM
C      CORX IS THE ADJUSTMENT FOR DX STREAK.
C           IF MORE THAN ONE BURST PER STREAK. CORX IS NOT UNITY
      CORX=1.
C     CORT IS THE TIME ADJUSTMENT FOR DT-STREAK
      CORT=1.
C     XD=DEVELOPMENT LENGTH (M)
C     XM=MODEL LENGTH (M)
      XD=3.5
      IMJ=0
      XM=0.0
```

```
  157  XD=XD+XM
       XM=0.0254
       XT=XD+XM
C      DXT=NODE SPACING ON MODEL (CM)
       DXT=1.27
       DXT=DXT/100.
       NXM=XM/DXT+0.999
C      XSMX IS THE RIGHT-MOST LOCATION STORING PRESSURE
       XSMX=XD+(NXM-0.5)*DXT
C      US=FREE STREAM VELOCITY (M/SEC)
       US=15.2
C      BLT=BOUNDARY LAYER THICKNESS (CM)
       BLT=2.54
C      DPT=DISPLACEMENT THICKNESS--CALCULATED USING 1/7 POWER LAW UNLESS
C           OTHERWISE SPECIFIED
       DPT=7*BLT/72
C      ENTER OTHER DISPLACEMENT THICKNESS HERE IF DESIRED (CM)
       DPT=DPT
       BLT=BLT/100.
       DPT=DPT/100.
C      UFRIC=FRICTION VELOCITY (M/SEC)
       UFRIC=1.08
C      RHO=AIR DENSITY (KG/CU.METER)
```

```
      RHO=1.2
C     CNU=KINEMATIC VISCOSITY OF AIR (SQ.METERS/SEC)
      CNU=0.000015
C     CALCULATION OF WALL SHEAR STRESS. TW
      TW=RHO*UFRIC*UFRIC
C     CALCULATION OF RMS PRESSURE FLUCTUATION
      PRMS=3.*TW
C     MISCELLANEOUS CONSTANTS
      PI=3.1415926
      TPI=2.*PI
      HPI=PI/2.
      C0=2.515517
      C1=0.802853
      C2=0.010328
      D1=1.432788
      D2=0.189269
      D3=0.001308
C     STARTER FOR RANDOM NUMBER GENERATOR
      XSTART=77653.
      RNM=URAN(XSTART)
      XSTART=0.0
C     CALCULATION OF NOMINAL PEAK FREQUENCY
      FPEAK=0.20574*US/(TPI*DPT)
```

```
      FMAX=10.*FPEAK
C     COMPATABLE TIME STEP
      DTT=1./FMAX/10.
C     A USER SPECIFIED TIME STEP CAN BE ENTERED HERE
C     THIS TIME STEP IS THE TIME INCREMENT USED IN THE RESOLUTION OF THE
C          OUTPUT--THE INTERNAL FLUCTUATION TIME STEP IS RANDOM
C     MAXIMUM TIME IS CONSTRAINED BY COMPUTER STORAGE.  SET THE NUMBER
C          OF ALLOWABLE TIMESTEPS--NTM
      NTM=10000
C     INITIAL TIME IS TO(SEC)
      TO=0.
      DTAVG=0.
C     PREFIXES FOR RANDOM NUMBER CALCULATIONS
      PX=DPT*UFRIC/US
      PT=DPT/US
      PW=US/DPT
C     CALCULATION OF REQUIRED START UP TIME FOR SIMULATION
      TSO=1.8*XT/US
      NMIN=TSO/DTT
      TMAX1=NMIN*DTT
C     IF NMIN IS GREATER THAN NTM, NTM IS OVERRIDDEN
      IF(NMIN-NTM)201,201,202
  202 NTM=NMIN
```

```
  201 CONTINUE
      TMAX=NTM*DTT
      WRITE(6,100) DTT,TMAX
  100 FORMAT(5X,*DT=*,F10.8,10X,*TMAX=*,F10.4,//)
      TMAXS=CDIM*DTT
      TMAX=TMAX+TMAX1
      NTM2=NDIM-NMIN+2
      TREF=TMAX1
      NREL=0
C     INITIALIZE PRESSURE ARRAY
      DO 52 NX=1,NXM
      DO 52 NT=1,NDIM
   52 P(NX,NT)=0.0
      NFLG=0
      TSUB=0.
C     INITIALIZE LOCATION AND TIME BASE, ETC.
    1 X=0.
      NCT=0
      DTSUM=0.
      TO=TO+DTAVG
      IF(TO-TMAX1)2,150,150
  150 IF(NFLG) 151,151,15
  151 NFLG=1
```

```
      TMAX1=TMAX
      DO 154 I=1.NXM
      DO 152 J=NMIN.NDIM
      JJ=J+1-NMIN
      P(I.JJ)=P(I.J)
  152 CONTINUE
      DO 153 J=NTM2.NDIM
      P(I.J)=0.0
  153 CONTINUE
  154 CONTINUE
      TSUB=TREF
      TMAXS=TMAXS+TREF
      NREL=NMIN
    2 RNM=URAN(XSTART)
      RNM=0.005+0.99*RNM
C     CALCULATION OF DX USING RANDOM NUMBER RNM
      HPII=HPI*RNM
      DX=PX*(32.2-2/(RNM+0.619)+72.*RNM**2+0.63*TAN(HPII))
      DX=CORX*DX
      X=X+DX
      RNM=URAN(XSTART)
      RNM=0.005+0.99*RNM
C     CALCULATION OF RADIAN FREQ. FROM NEW RNM
```

```
      SRNM=SQRT(RNM)
      RNMS=RNM*RNM
      RRNM=(1.-RNM)**0.25
      FRNM=1/RRNM-1
      W=PW*(0.2713*SRNM-0.3070*RNM+0.7899*RNMS+3.3518*FRNM)
      F=W/TPI
      TP=1./F
      DXE=0.8*US*TP
      XO=X-DXE
C     XO IS THE ORIGIN OF THE SINE WAVE FLUCTUATION
C     X IS THE FRONT OF THE SINE WAVE
C     CHECK TO SEE IF THE DISTURBANCE IS OVER THE MODEL
      IF(X-XD)5,5,3
C     IF THE DISTURBANCE IS OVER THE MODEL, HAS IT PASSED THE LAST DATA
C         STATION
    3 IF(XO-XSMX)4,1,1
C     NSTORE IS USED TO FLAG CALCULATIONS WHEN DISTURBANCE IS OVER THE
C         MODEL
    4 NSTORE=0
      IF(XO-XD)5,5,50
   50 NXI=(XO-XD)/DXT+1.5
      GO TO 6
    5 NSTORE=1
```

```
      NXI=1
C     GENERATION OF RANDOM TIME STEP
    6 RNM=URAN(XSTART)
      RNM=0.005+0.99*RNM
      HPII=HPI*RNM
      DT=PT*(32.2-2./(RNM+0.619)+72.*RNM**2+0.63*TAN(HPII))
      DT=CORT*DT
      T=TO+DT
      NCT=NCT+1
      DTSUM=DTSUM+DT
      DTAVG=DTSUM/NCT
C     GENERATION OF GAUSSIAN RANDOM PRESSURE AMPLITUDE
      RNM=URAN(XSTART)
      CIND=RNM+0.5
      IND=CIND
      CIND=IND
      PPP=2.*(1.-CIND)-1.
      ARGR=RNM/(1.+CIND)
      ARG=1./(ARGR*ARGR)
      CT=ALOG(ARG)
      CM=SQRT(CT)
      PMG=CM-(CO+CM*(C1+CM*C2))/(1+CM*(D1+CM*(D2+CM*D3)))
      PE=PRMS*PMG*PPP
```

```
C     DO LOOP FOR STEPPING THROUGH MODEL STORAGE LOCATIONS
      DO 14 NX=NX1,NXM
C     MODEL STATION X-LOCATION
      AX=NX
      XS=XD+(AX-.5)*DXT
C     ARRIVAL TIME OF PRESSURE FLUCTUATION
      DXS=XS-X
      TGO=1.25*DXS/US+T
C     FLUCTUATION DEPARTURE TIME
      DXO=XS-XO
      TSTP=1.25*DXO/US+T
C     DOES TSTP EXCEED TMAX
      IF(TSTP-TMAXS)9,9,7
    7 IF(TGO-TMAXS)8,8,13
    8 NSTOP=NDIM
      GO TO 10
    9 NSTOP=TSTP/DTT
   10 NGO=TGO/DTT
      IF(NGO-NREL)99,99,98
   98 CONTINUE
C     DO LOOP FOR SUCCESSIVE TIME CONTRIBUTIONS TO THE SAME X LOCATION
      DO 12 NT=NGO,NSTOP
      TC=NT*DTT
```

```
      THET=TPI*(TC-TGO)/(TSTP-TGO)
      DELT=TC-T
      XOT=0.8*US*DELT
      IF(XOT-0.0005)19,19,18
   18 CONTINUE
      ARGX=-4267*CNU/(XOT*UFRIC)
      DECA=1-EXP(ARGX)
      GO TO 23
   19 CONTINUE
      DECA=1.
   23 CONTINUE
      DP=PE*SIN(THET)*DECA
      NTIME=NT-NREL
      P(NX,NTIME)=P(NX,NTIME)+DP
   12 CONTINUE
   99 CONTINUE
   13 CONTINUE
   14 CONTINUE
  130 FORMAT(5X,F10.6,6E14.6)
      GO TO 2
   15 CONTINUE
      NWRT=NTM/8
      GO TO 1671
```

```
      DO 160 I=1,NWRT
      I1=(I-1)*8+1
      I2=I1+7
      WRITE(6,161)I1,(P(1,J),J=I1,I2)
  161 FORMAT(5X,*N=*,I4,2X,8E14.8)
      WRITF(6,162) (P(2,J),J=I1,I2)
  162 FORMAT(16X,8E14.8,/)
  160 CONTINUE
 1671 CONTINUE
      CNTM=NTM
      P1A=0.
      P1RMS=0.
      DO 170 IN=1,NTM
      P1A=P1A+P(1,IN)
      P1RMS=P1RMS+P(1,IN)**2
  170 CONTINUE
      P1A=P1A/CNTM
      P1RMS=(P1RMS/CNTM)**0.5
      WRITE(6,171) P1A,P1RMS
  171 FORMAT(20X,*P1AVG=*,E14.8,5X,*P1RMS=*,E14.8)
      WRITE(1)((P(K,KK),KK=1,NTM),K=1,NXM)
      IMJ=IMJ+1
      IF(IMJ.LT.11)GO TO 157
```

```
      STOP
      END
```